

Dynamics of interacting dark energy model in Einstein and Loop Quantum Cosmology

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Abstract

We investigate the background dynamics when dark energy is coupled to dark matter in the universe described by Einstein cosmology and Loop Quantum Cosmology. We introduce a new general form of dark sector coupling, which presents us a more complicated dynamical phase space. Differences in the phase space in obtaining the accelerated scaling attractor in Einstein cosmology and Loop Quantum Cosmology are disclosed.

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I. INTRODUCTION

Our universe is undoubtedly undergoing an accelerated expansion driven by a yet unknown dark energy (DE)[1, 2, 3, 4]. This mysterious energy component occupies almost 70% of the content of the universe today. The leading interpretation of such a DE is a cosmological constant with equation of state (EoS) $\omega = -1$. Although this interpretation is consistent with observational data, at the fundamental level it fails to be convincing. The vacuum energy density is far below the value predicted by any sensible quantum field theory, and it suffers the coincidence problem, namely, “why are the vacuum and matter energy densities of precisely the same order today?”. To overcome the coincidence problem, some sophisticated dynamical DE models relating the DE to scalar fields have been put forward to replace the cosmological constant [5].

Considering that DE contributes a significant fraction of the content of the universe, it is natural to look into its interaction with the remaining fields of the Standard Model in the framework of field theory. The possibility that DE and dark matter (DM) can interact has got growing attention recently [6]-[21]. It has been argued that an appropriate interaction between DE and DM can influence the perturbation dynamics and affect the lowest multipoles of the CMB spectrum [9, 12]. Recently, it has been shown that such a coupling can be inferred from the expansion history of the Universe, as manifested in the supernova data together with CMB and large-scale structure [13]. Signatures of the interaction between DE and DM in the dynamics of galaxy clusters has also been analyzed [14, 15]. It has been argued that the coupling between DE and DM can provide a mechanism to alleviate the coincidence problem and lead to an accelerated scaling attractor solution with similar energy densities in the dark sectors today [7, 11].

A general interaction between DE and DM can be described in the background by the balance equations

$$\begin{aligned}\dot{\rho}_x + 3H(1 + \omega_x)\rho_x &= -\Gamma, \\ \dot{\rho}_m + 3H\rho_m &= \Gamma,\end{aligned}\tag{1}$$

where ρ_m and ρ_x correspond to the energy densities of DM and DE, respectively. Here Γ describes the coupling between DE and DM. Since the nature of dark sectors remain unknown, there is as yet no basis in fundamental theory for a specific coupling in the dark sectors. All coupling models discussed at the present moment are necessarily phenomenological [6]. There are two criterions to determine whether some models can be more physical justification than the others. One is to confront observations. The other is to examine whether the coupling can lead to accelerated scaling attractor solutions[16], which is a decisive way to achieve similar energy

densities in dark sectors and alleviate the coincidence problem. In this work we introduce a new form of dark sector coupling, $\Gamma = 3cH\rho_x^\alpha\rho_m^{1-\alpha}$. This model is more general than the coupling discussed in the literatures. When $\alpha = 0, 1$, it reduces to the cases with coupling between dark sectors solely proportional to the energy densities of DM [17] and DE [18], respectively. These two limiting cases have been examined thoroughly against observations and their possibilities to alleviate coincidence problem have also been discussed [19, 20]. Here we will investigate the background dynamics when the DE is coupled to DM via this general interaction form. We will show that the general coupling leads to a more complicated dynamical phase space.

Besides the discussion of the dynamics of DE with our general coupling to DM in the universe described by the Einstein theory, we will also extend our investigation to the Loop Quantum Cosmology (LQC). The LQC [22, 23, 24] is the application of the Loop Quantum Gravity [25, 26, 27] in the cosmological context, which keeps the properties of non-perturbative and background independent quantization of gravity. Recent investigations have shown that the loop quantum effects can be very well described by an effective modified Friedmann dynamics. There are two types of modification to the Friedmann equation. The first one is based on the modification to the behavior of inverse scale factor operator below a critical scale factor a_* . Considering these modifications one can obtain many interesting results including the replacement of the classical big bang by a quantum bounce with desirable features [28], avoidance of many singularities [29], easier inflation [30], and so on. However, the first type of modification to Friedmann equation suffers from gauge dependence which can not be cured and thus yields unphysical effects. The second type of modification to Friedmann equation is discovered very recently. It adds a $-\rho^2/\rho_c$ term in the standard Friedmann equation which essentially encodes the discrete quantum geometric nature of spacetime [24, 31, 32]. When energy density of the universe becomes of the same order of a critical density ρ_c , this modification becomes dominant and the universe begins to bounce and then oscillates forever. Thus the big bang singularity, the big rip and other future singularities at semi-classical regime can be avoided in LQC [24, 31, 34, 35, 36]. Therefore by using the second type of modification to Friedmann equation, the physically appealing features of the first type are retained. For the universe with a large scale factor, the first type of modification to the effective Friedmann equation can be neglected and only the second type of modification is important. Thus the dynamics of DE (phantom [35, 37, 38, 39], quintom and hessence [40]) have been investigated recently in LQC on the basis of the second type of modification. It finds that the dynamical properties of dark energy models in LQC have different behaviors from those in the classical Einstein cosmology. Here we will examine the background dynamics of

the LQC dominated by DE and DM where there is the general coupling Γ between dark sectors and compare the results with those in Einstein cosmology.

The paper is organized as follows: in sections II and III, we study the dynamics of the interacting dark energy model in Einstein Cosmology and the LQC, respectively. In Sec.IV, we present numerical pictures of dynamics in Einstein cosmology and LQC. Our conclusions and discussions will be presented in the last section.

II. DYNAMICS OF THE INTERACTING DARK ENERGY MODEL IN EINSTEIN COSMOLOGY

In the Einstein theory, the universe is described by the standard Friedmann equation

$$H^2 = \frac{\kappa}{3}\rho, \quad (2)$$

where H is the Hubble parameter, $\rho = \rho_m + \rho_x$ is the total energy density and the constant $\kappa = 8\pi G$. Since we are concentrating on the late time accelerating universe, we have neglected the radiation and baryons for simplicity.

Differentiating Eq.(2) and using the conservation law of the total energy $\dot{\rho} + 3H(\rho + p) = 0$, we have

$$\dot{H} = -\frac{\kappa}{2}(\rho + p). \quad (3)$$

To analyze the evolution of the dynamical system, we introduce the dimensionless variables

$$u \equiv \frac{\sqrt{\kappa\rho_x}}{\sqrt{3}H}, \quad v \equiv \frac{\sqrt{\kappa\rho_m}}{\sqrt{3}H}, \quad \frac{d}{dN} = \frac{1}{H} \frac{d}{dt}, \quad (4)$$

where $N \equiv \ln a$ is the number of e -folding to represent the cosmological time. Using the above definitions, the Hubble equations can be rewritten as

$$u^2 + v^2 = 1, \quad (5)$$

and

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \left[1 + \frac{\omega_x u^2}{u^2 + v^2} \right] = -\frac{3}{2} (1 + \omega_x u^2). \quad (6)$$

The effective total EOS ω_{tot} is given by

$$\omega_{tot} = \frac{\omega_x \rho_x}{\rho_x + \rho_m} = \frac{\omega_x u^2}{u^2 + v^2} = \omega_x u^2. \quad (7)$$

Using dimensionless variables, the dynamical equations of the system can be expressed as

$$\begin{aligned} u' &= \frac{3u}{2} \left[\omega_x (u^2 - 1) - c \left(\frac{u^2}{v^2} \right)^{\alpha-1} \right], \\ v' &= \frac{3v}{2} \left[\omega_x u^2 + c \left(\frac{u^2}{v^2} \right)^{\alpha} \right], \end{aligned} \quad (8)$$

where the prime denotes a derivative with respect to N . The critical points u_c, v_c satisfy $u' = 0$ and $v' = 0$. In order to study the stability of the critical points, we expand about the critical points $u = u_c + \delta u, v = v_c + \delta v$ and linearize the above equations near the critical points so that

$$\begin{aligned} \delta u' &= \frac{3}{2} \left[3\omega_x u_c^2 - 1 - (2\alpha - 1)c \left(\frac{u_c}{v_c} \right)^{2\alpha-2} \right] \delta u + \left[3c(\alpha - 1) \left(\frac{u_c}{v_c} \right)^{2\alpha-1} \right] \delta v, \\ \delta v' &= \left[3\omega_x u_c v_c + 3c\alpha \left(\frac{u_c}{v_c} \right)^{2\alpha-1} \right] \delta u + \frac{3}{2} \left[\omega_x u_c^2 - (2\alpha - 1)c \left(\frac{u_c}{v_c} \right)^{2\alpha} \right] \delta v. \end{aligned} \quad (9)$$

The eigenvalues of the matrix of coefficients of the above equations encode the behavior of the dynamical system near the critical points.

In general, for an arbitrary α it is difficult to obtain the analytical forms of the critical points. Here we only consider some specific values of α and examine the dynamics of DE with an interaction with DM.

A. Case I: $\alpha = 0$

When $\alpha = 0$, the interaction form reduces to the simple coupling between DE and DM in proportional to the energy density of DM. This simple interaction form has been confronted to observations and its possibility to alleviate the coincidence problem has also been examined [19]. From the dynamical system equations, we can obtain two critical points for the specific model, namely:

$$\begin{aligned} &\bullet \text{ Point } A_0 : (1, 0), \\ &\bullet \text{ Point } B_0 : \left(\sqrt{-\frac{c}{\omega_x}}, \sqrt{1 + \frac{c}{\omega_x}} \right). \end{aligned} \quad (10)$$

The eigenvalues of the coefficient matrix of the linearized equations around these critical points can be expressed respectively as

$$\begin{aligned} &\bullet \text{ Point } A_0 : \quad \lambda_1 = 3\omega_x, \quad \lambda_2 = \frac{3}{2}(c + \omega_x), \\ &\bullet \text{ Point } B_0 : \quad \lambda_1 = -3c, \quad \lambda_2 = -3(c + \omega_x). \end{aligned} \quad (11)$$

For point A_0 , when $c < -\omega_x$, both eigenvalues λ_1 and λ_2 are negative, which indicates that A_0 is a stable point. From Eq.(7), we learn that the effective total EOS at point A_0 is $\omega_{tot} = \omega_x$. Therefore we obtain

$\ddot{a} \propto -(1 + 3\omega_x) t^{\frac{2}{3(1+\omega_x)}-2}$ and $\rho \propto a^{-3(1+\omega_x)}$. It means that point A_0 is an accelerated scaling solution as $\omega_x < -1/3$ and there is singularity in the finite future as $\omega_x < -1$. When $c > -\omega_x$, the sign of λ_1 is always opposite to the sign of λ_2 , which leads A_0 to a saddle point. For point B_0 , the critical point v_c exists only provided that $c \leq -\omega_x$, which leads the sign of λ_1 always opposite to that of λ_2 . Thus point B_0 is a saddle point.

B. Case II: $\alpha = 1$

In this limiting case the coupling between dark sectors is in proportional to the energy density of DE. This simple coupling has been examined using observational data and its effect to alleviate the coincidence problem has been discussed [19]. One can obtain two critical points in this dynamical system:

- Point $A_1 : (0, 1)$,
- Point $B_1 : \left(\sqrt{1 + \frac{c}{\omega_x}}, \sqrt{-\frac{c}{\omega_x}} \right)$.

(12)

The eigenvalues of the coefficient matrix of the linearized equations are

- Point $A_1 : \quad \lambda_1 = 0, \quad \lambda_2 = -\frac{3}{2}(c + \omega_x)$,
- Point $B_1 : \quad \lambda_1 = 3(c + \omega_x), \quad \lambda_2 = 3(c + \omega_x)$.

(13)

It is easy to examine that the critical point A_1 is not a stable point, while B_1 is stable when $c \leq -\omega_x$. The total effective EOS at point B_1 reads $\omega_{tot} = c + \omega_x$. When $\omega_x \leq -1/3 - c$, point B_1 can be an accelerated scaling solution. From Eq.(6) we find $H = \frac{2}{3(1+\omega_x+c)(t_0-t)}$, $\dot{H} = -\frac{2}{3(1+\omega_x+c)(t_0-t)^2}$. Thus as $\omega_x \leq -1 - c$ the universe will undergo super-accelerated expansion ($\dot{H} > 0$) and end in the big rip.

C. Case III: $\alpha = \frac{1}{2}$

Solving equations $u' = 0$ and $v' = 0$, one can obtain two critical points of the dynamical system:

- Point $A_2 : \left(\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}}, \sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}} \right)$,
- Point $B_2 : \left(\sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}}, \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}} \right)$.

(14)

Through analysis of the eigenvalues of the coefficient matrix, we find that point A_2 is stable when $c < -\frac{\omega_x}{2}$, while point B_2 is a saddle point. At point A_2 , the total effective EOS is $\omega_{tot} = (\omega_x - \sqrt{\omega_x^2 - 4c^2})/2$, which

shows that if $c > \frac{1}{3}$ and $\omega_x < -2c$ or $c < \frac{1}{3}$ and $-\frac{2}{3} < \omega_x < -\frac{1}{3} - 3c^2$, we have $\omega_{tot} < -\frac{1}{3}$ so that A_2 is an accelerated scaling attractor. When $\omega_x < \frac{2}{3}(1 - 2\sqrt{1 + 3c^2})$, one obtains $\omega_{tot} < -1$, $\dot{H} > 0$ and finds there is a future singularity in this case.

D. Case IV: $\alpha = -1$

In this case the system has two critical points:

- Point $A_3 : (1, 0)$,
- Point $B_3 : \left(\sqrt{\frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}}, \sqrt{1 - \frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}} \right)$.

(15)

In term of the signs of the eigenvalues of the coefficient matrix, we find that point A_3 is stable when $\omega_x < 0$, however, point B_3 is a saddle point. From Eq.(7), we learn that point A_3 can be an accelerated scaling attractor provided that $\omega_x < -\frac{1}{3}$ and there is a future singularity as $\omega_x < -1$.

E. Case IV: $\alpha = 2$

When $\alpha = 2$, the system has two critical points:

- Point $A_4 : (0, 1)$,
- Point $B_4 : \left(\sqrt{1 - \frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}}, \sqrt{\frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}} \right)$.

(16)

Similarly, we find that point A_4 is unstable, while point B_4 is stable for all $\omega_x < 0$. The total effective EOS at point B_4 is $\omega_{tot} = \omega_x - (c - \sqrt{c^2 - 4\omega_x c})/2$. When $\omega_x < -\frac{1}{3}(1 + \sqrt{3c})$, we have $\omega_{tot} < -\frac{1}{3}$ and B_4 can be an accelerated scaling solution. Moreover, we find that $\omega_{tot} > -1$ as $\omega_x > -1 - \sqrt{c}$ and there is no future singularity in this case. But as $\omega_x < -1 - \sqrt{c}$ one can obtain $\omega_{tot} < -1$ and a future singularity is inevitable. Thus in the Einstein cosmology the presence of the coupling terms can not remove the singularity entirely.

III. DYNAMICS OF THE INTERACTING DARK ENERGY MODEL IN LQC

In this section we are going to extend our discussion to the LQC. The loop quantum effect modifies the Friedmann equation into [24, 31, 32, 33]

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \quad (17)$$

where $\rho_c \equiv \frac{\sqrt{3}}{16\pi^2\gamma^3 G^2 \hbar}$ is the critical loop quantum density and γ is the Barbero-Immirzi parameter. Let us note here it has been suggested that $\gamma \approx 0.2375$ by the black hole thermodynamics in LQG [34].

Differentiating Eq.(17) and using the conservation equation of the total energy density $\dot{\rho} + 3H(\rho + p) = 0$, where $\rho = \rho_x + \rho_m$, one can obtain

$$\dot{H} = -\frac{\kappa}{2}(\rho + p)\left(1 - \frac{2\rho}{\rho_c}\right). \quad (18)$$

Adopting the dimensionless variables defined in (4), the evolution of the Hubble parameter in LQC becomes

$$(u^2 + v^2)\left(1 - 3H^2 \frac{u^2 + v^2}{\rho_c}\right) = 1, \quad (19)$$

and

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(2 - u^2 - v^2)\left[1 + \frac{\omega_x u^2}{u^2 + v^2}\right]. \quad (20)$$

The total effective EOS ω_{tot} in LQC is given by

$$\omega_{tot} = \frac{\omega_x \rho_x}{\rho_x + \rho_m} = \frac{\omega_x u^2}{u^2 + v^2}. \quad (21)$$

The dynamical system can be expressed as

$$\begin{aligned} u' &= \frac{3u}{2}\left[-1 - \omega_x - c\left(\frac{u}{v}\right)^{2\alpha-2} + \frac{[(1 + \omega_x)u^2 + v^2](2 - u^2 - v^2)}{u^2 + v^2}\right], \\ v' &= \frac{3v}{2}\left[-1 + c\left(\frac{u}{v}\right)^{2\alpha} + \frac{[(1 + \omega_x)u^2 + v^2](2 - u^2 - v^2)}{u^2 + v^2}\right], \end{aligned} \quad (22)$$

Obtaining the critical points and linearizing the system near them, we can study the stability of critical points by analyzing the first-order differential equations

$$\begin{aligned} \delta u' &= \frac{3}{2}\left[-3(\omega_x + 1)u^2 - v^2 + 1 - \omega_x + \frac{2u^2(u^2 + 3v^2)}{(u^2 + v^2)^2} - (2\alpha - 1)c\left(\frac{u}{v}\right)^{2\alpha-2}\right]\delta u \\ &\quad - 3\left[uv + \frac{2\omega_x u^3 v}{(u^2 + v^2)^2} - (\alpha - 1)c\left(\frac{u}{v}\right)^{2\alpha-1}\right]\delta v, \\ \delta v' &= 3\left[-(\omega_x + 1)uv + \frac{2\omega_x u v^3}{(u^2 + v^2)^2} + \alpha c\left(\frac{u}{v}\right)^{2\alpha-1}\right]\delta u \\ &\quad + \frac{3}{2}\left[1 - \omega_x - u^2 - 3v^2 + \frac{2\omega_x u^2(u^2 - v^2)}{(u^2 + v^2)^2} - (2\alpha - 1)c\left(\frac{u}{v}\right)^{2\alpha}\right]\delta v. \end{aligned} \quad (23)$$

Solving the eigenvalues of the coefficient matrix of the above equations, we can know the behavior of the dynamical system near the critical points. Comparing with the Einstein theory, we find that the same critical points will have different eigenvalues in the coefficient matrix in LQC. This means that the dynamical property of the system in the LQC is different from that in the Einstein cosmology.

As did in section II, here we will also focus on some specific cases for simplicity, such as $\alpha = 0, 1, \frac{1}{2}, -1, 2$. For these specific cases, the critical points in the LQC are the same as those in the Einstein cosmology.

A. Case I: $\alpha = 0$

In this limiting case, the critical points of the dynamical system are

- Point $A_0 : (1, 0)$,
- Point $B_0 : \left(\sqrt{-\frac{c}{\omega_x}}, \sqrt{1 + \frac{c}{\omega_x}} \right)$.

(24)

The eigenvalues of the linearized equation around the critical points read

- Point $A_0 : \quad \lambda_1 = -3(1 + \omega_x), \quad \lambda_2 = \frac{3}{2}(c + \omega_x)$,
- Point $B_0 : \quad \lambda_1 = 3(c - 1), \quad \lambda_2 = -3(c + \omega_x)$.

(25)

It is easy to see that for the DE with $\omega_x > -1$ point A_0 is stable if $c < -\omega_x$. The effective total EOS $\omega_{tot} = \omega_x$, thus the critical point A_0 is an accelerated scaling solution without a future singularity for $-1 < \omega_x < -1/3$. When the DE is phantom like $\omega_x < -1$, A_0 is a saddle point. This is different from that in the Einstein theory, where A_0 is still stable for the phantom DE provided that $c < -\omega_x$. Point B_0 is a saddle point if $c < 1$ and is unstable when $c > 1$. In the Einstein theory, B_0 is always a saddle points. This means that the presence of the term $-\frac{\rho}{\rho_c}$ in the Friedmann equation due to the quantum correction changes the dynamical properties of autonomous system. Moreover, when the value of ω_x goes beyond the ranges of $-1 < \omega_x < -1/3$, the quantum bounce originated from the term $-\frac{\rho}{\rho_c}$ will leads to the avoidance of the future singularity.

B. Case II: $\alpha = 1$

The critical points are the same as those in the Einstein theory

- Point $A_1 : (0, 1)$,
- Point $B_1 : \left(\sqrt{1 + \frac{c}{\omega_x}}, \sqrt{-\frac{c}{\omega_x}} \right)$.

(26)

However, the eigenvalues of the coefficient matrix of the linearized equations near the critical points become

- Point $A_1 : \quad \lambda_1 = -3, \quad \lambda_2 = -\frac{3}{2}(c + \omega_x)$,
- Point $B_1 : \quad \lambda_1 = 3(c + \omega_x), \quad \lambda_2 = -3(1 + c + \omega_x)$.

(27)

Point A_1 is a stable point when $c > -\omega_x$. From Eq. (20) we have $\omega_{tot} = 0$ at the point A_1 , which means that in this model our universe described by LQC will enter DM dominated era and there is no singularity in the finite future. However, in the Einstein theory A_1 is unstable. Point B_1 can be stable provided that $-(1 + \omega_x) < c < -\omega_x$. This region of c to keep B_1 stable is smaller than that in the Einstein theory. At point B_1 , we have $\omega_{tot} = c + \omega_x$, which shows that when $-1 - c < \omega_x < -\frac{1}{3} - c$, we have $-1 < \omega_{tot} < -\frac{1}{3}$, so that point B_1 corresponds to an accelerated attractor without a future singularity.

C. Case III: $\alpha = \frac{1}{2}$

The critical points can be found at

$$\begin{aligned} \bullet \text{ Point } A_2 : & \left(\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}}, \sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}} \right), \\ \bullet \text{ Point } B_2 : & \left(\sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}}, \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c^2}{\omega_x^2}}} \right). \end{aligned} \quad (28)$$

Analyzing the stability, we find that when DE is of quintessence type, point A_2 is stable provided that $c < -\frac{\omega_x}{2}$. When the DE is of phantom type, A_2 can be stable only when $\sqrt{-(1 + \omega_x)} < c < -\frac{\omega_x}{2}$ and $\omega_x > -2$. From the total effective EOS, we learn that when $c > \frac{1}{3}$ and $-1 - c^2 < \omega_x < -2c$ or $c < \frac{1}{3}$ and $-1 - c^2 < \omega_x < -\frac{1}{3} - 3c^2$, point A_2 is an accelerated attractor. Similarly, when the value of ω_x goes beyond the ranges above, the effects from the term $-\frac{\rho}{\rho_c}$ will make the future singularity disappear. Point B_2 is a saddle point, which agrees with that found in the Einstein theory.

D. Case IV: $\alpha = -1$

Critical points of the system read

$$\begin{aligned} \bullet \text{ Point } A_3 : & (1, 0), \\ \bullet \text{ Point } B_3 : & \left(\sqrt{\frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}}, \sqrt{1 - \frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}} \right). \end{aligned} \quad (29)$$

For the quintessence type DE, point A_3 is a stable point. Since $\omega_{tot} = \omega_x$, A_3 is an accelerated scaling attractor. For the phantom type DE, A_3 is a saddle point when $c < -\frac{1}{1+\omega_x}$ and is unstable when $c > -\frac{1}{1+\omega_x}$.

Point B_3 is a saddle point always.

E. Case IV: $\alpha = 2$

When $\alpha = 2$, the system has two critical points:

$$\begin{aligned} &\bullet \text{ Point } A_4 : (0, -1), \\ &\bullet \text{ Point } B_4 : \left(\sqrt{1 - \frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}}, \sqrt{\frac{c - \sqrt{c^2 - 4\omega_x c}}{2\omega_x}} \right). \end{aligned} \quad (30)$$

Similarly, we find that point A_4 is a saddle point as that in the Einstein theory. When the DE is of quintessence type, point B_4 is stable and when $-1 < \omega_x < -\frac{1}{3}(1 + \sqrt{3c})$ B_4 is an accelerated scaling attractor. When the DE is of phantom type, B_4 can be stable only when $c > (1 + \omega_x)^2$. When $-(1 + \sqrt{c}) < \omega_x < -\frac{1}{3}(1 + \sqrt{3c})$, B_4 can be an accelerated scaling solution as well. Although $\omega_{tot} < -1$ as $\omega_x < -1 - \sqrt{c}$, the loop quantum effects will cancel off the future singularity in the evolution of the universe. Thus in the LQC the big rip can be removed entirely.

IV. NUMERICAL RESULTS

In this section we confirm numerically the complicated stability conditions for critical points obtained above. For the new general form of the interaction between DE and DM, $\Gamma = 3cH\rho_x^\alpha\rho_m^{1-\alpha}$, we have more complicated dynamical phase spaces. We find that the position of the critical point and its stability depend not only on the coupling constant c , the DE EOS ω_x and the exponent α in the coupling, but also on the theory to describe the universe. In figure (1), we show the stable regions in the parameter space (c, ω_x) by choosing $\alpha = 0.5$ and 2. In Einstein cosmology the critical points A_2 ($\alpha = 0.5$) and B_4 ($\alpha = 2$) are late time attractor in the region $I + II$. However in LQC, A_2 and B_4 are late time attractors only in the region II . In LQC we see that the region of the location of the accelerated scaling attractor has been reduced compared to the Einstein theory. This holds true for other values of α .

In figure (2), we plot the numerical result to illustrate the phase space trajectories for our coupling model with chosen c and ω_x in the stable region and selected α in the Einstein cosmology and the LQC. Since the LQC has the same critical points as that in the Einstein cosmology, we see that for fixed α , all trajectories with different initial conditions in the Einstein cosmology and LQC converge to the same final state determined by parameters c and ω_x . This means that our universe will enter an era with similar energy densities of DE and DM.

In figure (3), we show the evolution of the effective EOS $\omega_{tot} = \omega_x\rho_x/(\rho_m + \rho_x)$ in the stable region. We

see that in the final state the total state parameter ω_{tot} tends to a constant, which is determined by values of ρ_{x_c} , ρ_{m_c} , ω_x and α . For selected values of ω_x, c to be within a certain range discussed in sections II and III for different values of α , such as $\omega_x = -0.6$ and $c = 0.18$, we can have stable critical point and meanwhile we can get the total effective EOS $\omega_{tot} < -\frac{1}{3}$ in the final state as displayed in figure (3). Our universe will enter a final state with a constant energy ratio between DE and DM and accelerate forever for all chosen α s. However when values of ω_x, c are beyond the range discussed in Einstein cosmology and LQC, our universe will enter a decelerated expansion.

In figure (4) we exhibit the evolution of $\rho(t)$ for chosen $\alpha = -1$, $\rho_c = 0.82$ and parameters (ω_x, c) in the unstable region in LQC. We find that the universe finally enters an oscillating regime in the LQC. The oscillating frequencies of $\rho(t)$ depend on the coupling constant c and DE EOS ω_x . This oscillation behavior makes the universe experience bouncing, which can avoid singularity faced in the usual Einstein cosmology. This property also holds for other values of α in LQC.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have studied the background dynamics when DE is modelled coupling with DM via a new general form $3cH\rho_x^\alpha\rho_m^{1-\alpha}$ in Einstein cosmology and LQC. For selected values of α , we have examined stability behaviors of critical points and found accelerated scaling solutions to account for the similar energy densities in dark sectors today. In LQC, the parameter space for the existence of the accelerated scaling attractor is found smaller than that in the Einstein cosmology. In the unstable region, the universe described by the LQC will enter an oscillatory regime which can help to avoid the singularity usually met in Einstein cosmology.

The background dynamics for the new general form of dark sector coupling leads to more complicated features in dynamical phase space. In order to confront this model with observations, the cosmological perturbations with this coupling form need to be disclosed. Some discussions in this direction has been addressed recently in [41].

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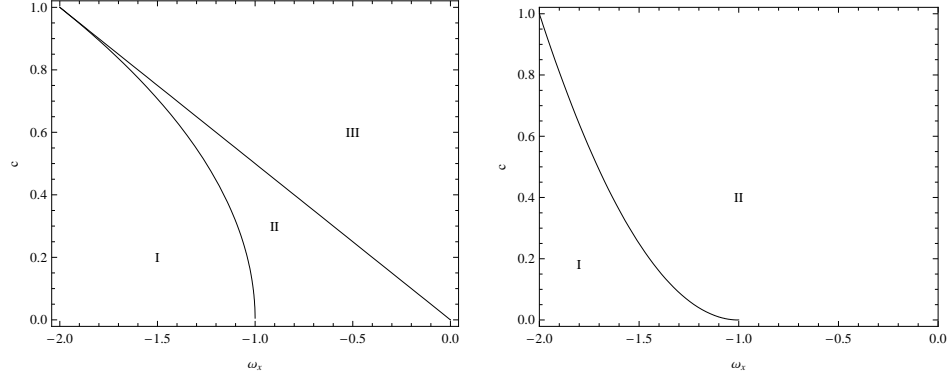


FIG. 1: The stable regions in the (ω_x, c) parameter space for fixed α (the left for $\alpha = 0.5$ and the right for $\alpha = 2$). In Einstein cosmology the critical points A_2 ($\alpha = 0.5$) and B_4 ($\alpha = 2$) are late time attractor in the region I+II. But in LQC, A_2 ($\alpha = 0.5$) and B_4 are late time attractor only in the region II. The region *III* represents the region of the solution without physical meaning.

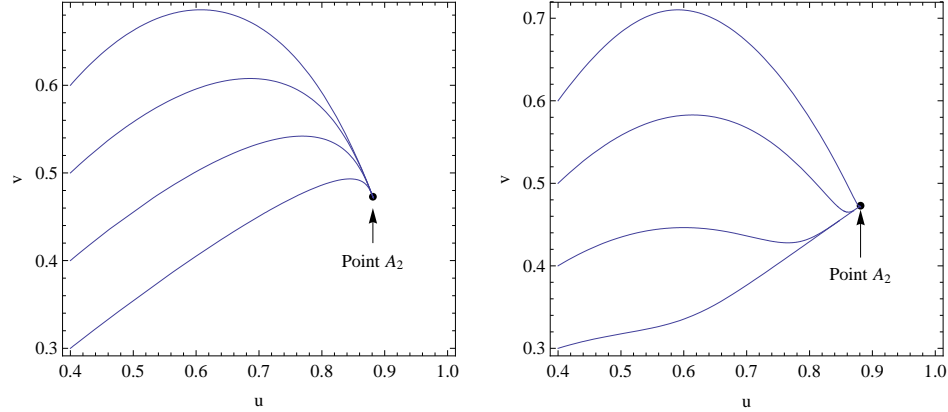


FIG. 2: The phase diagram of interacting dark energy (the left for Einstein cosmology and the right for LQC) with $\omega_x = -1.2$, $c = 0.5$ and $\alpha = 0.5$. The point A_2 is the critical point.

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- [1] A. G. Riess et al, *Astron. J.* **116**, 1009 (1998); P. de Bernardis et. al., *Nature* **404**, 955 (2000); A. G. Riess, et al., *astro-ph/0611572*.
 - [2] S. Perlmutter et al, *Astrophys. J.* **517**, 565 (1999); *Astrophys. J.* **598**, 102 (2003).
 - [3] D. J. Eisenstein, et al., *Astrophys. J.* **633**, 560 (2005).
 - [4] D. N. Spergel et al., *Astrophys. J. Suppl.* **170**, 377 (2007).

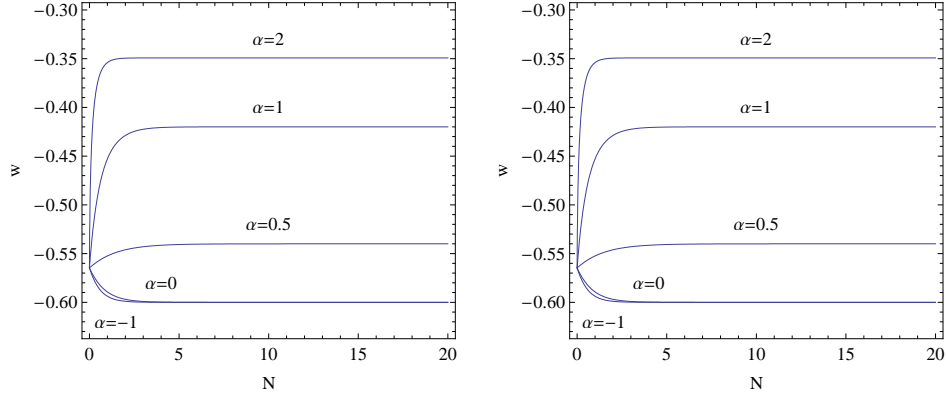


FIG. 3: The evolution of total cosmic energy ω for $\omega_x = -0.6$, $c = 0.18$ and fixed α (the left for Einstein cosmology, and the right for LQC).

- [5] T. Padmanabhan, Phys. Rept. **380**, 235 (2003); P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); V. Sahni, Lect Notes Phys. **653**: 141 (2004) and references therein.
- [6] W. Zimdahl, D. Pavon, L.P. Chimento, Phys. Lett. B **521**, 133 (2001) ; L.P. Chimento, A. S. Jakubi, D. Pavon, W. Zimdahl, Phys. Rev. D **67**, 083513 (2003); S. del Campo, R. Herrera and D. Pavon, Phys. Rev. D **70**, 043540 (2004); D. Pavon and W. Zimdahl, Phys. Lett. B **628**, 206 (2005).
- [7] G. Olivares, F. Atrio-Barandela and D. Pavon, Phys. Rev. D **71**, 063523 (2005); ibid. Phys. Rev. D **74**, 043521 (2006).
- [8] B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B **624**, 141 (2005); B. Wang, Ch.-Y. Lin, Elcio Abdalla, Phys. Lett. B **637**, 357(2006).
- [9] B. Wang, J. Zang, Ch.-Y. Lin, E. Abdalla and S. Micheletti, Nucl. Phys. B **778**, 69 (2007).
- [10] S. Das, P.S. Corasaniti and J. Khoury, Phys. Rev. D **73**, 083509 (2006).
- [11] L. Amendola, Phys. Rev. D **62**, 042511 (2000); L. Amendola, D. Tocchini-Valentini, Phys. Rev. D **64**, 043509 (2001); L. Amendola, S. Tsujikawa and M. Sami, Phys. Lett. B **632**, 155 (2006); L. Amendola and C. Quercellini, Phys. Rev. D **68**, 023514 (2003); G. W. Anderson and S. M. Carroll, astro-ph/9711288.
- [12] W. Zimdahl, Int. J. Mod. Phys. D **14**, 2319 (2005).
- [13] C. Feng, B. Wang, Y. Gong, R.-K. Su, JCAP **09**, 005 (2007).
- [14] O. Bertolami, F. Gil Pedro and M. Le Delliou, Phys. Lett. B **654**, 165 (2007). O. Bertolami, F. Gil Pedro and M. Le Delliou, arXiv:0705.3118v1.
- [15] E. Abdalla, L.Raul W. Abramo, L. Sodre Jr., B. Wang, arXiv:0710.1198 [astro-ph].
- [16] C. G. Bohmer, G. C. Cabral, R. Lazkoz, R. Maartens, Phys.Rev.**D78**, 023505,(2008).
- [17] Z. K. Guo, N. Ohta and S. Tsujikawa, Phys. Rev. D **76**, 023508 (2007); L. Amendola, G. Campos, R. Rosenfeld, astro-ph/0610806.
- [18] D. Pavon, B. Wang, arXiv:0712.0565.
- [19] J. He and B. Wang, JCAP 0806, 010, (2008); C. Feng, B. Wang, E. Abdalla, R.K. Su, Phys.Lett.**B 665** 111, (2008).
- [20] G. Olivares, F. Atrio-Barandela and D. Pavon, Phys. Rev. D **77**, 063513 (2008).
- [21] M. Manera and D. F. Mota, Mon. Not. Roy. Astron. Soc. **371**, 1373 (2006),(astro-ph/0504519); N. J. Nunes, and

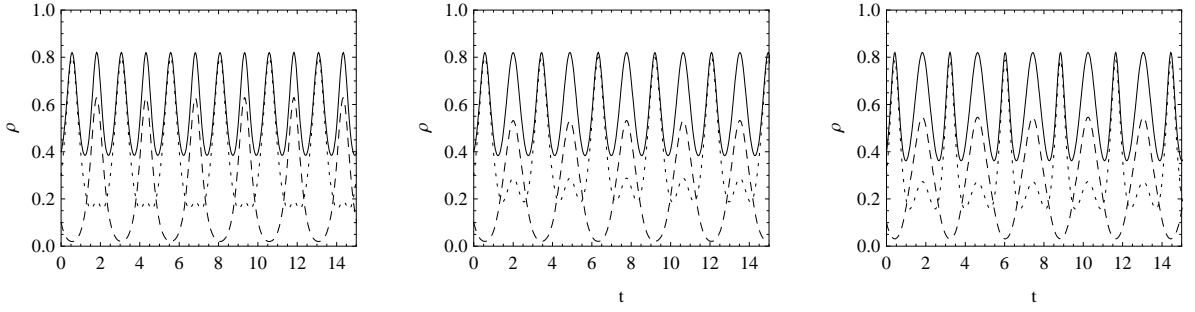


FIG. 4: The evolution of ρ with t in LQC for fixed $\alpha = -1$ and $\rho_c = 0.82$ (the left for $\omega_x = -1.6$ and $c = 0.1$, the middle for $\omega_x = -1.6$ and $c = 0.3$, and the right for $\omega_x = -1.8$ and $c = 0.3$). The solid, dotted and dashed curves correspond to $\rho_x + \rho_m$, ρ_x and ρ_m , respectively.

D. F. Mota, Mon. Not. Roy. Astron. Soc. **368**, 751 (2006), (astro-ph/0409481).

[22] M. Bojowald, Living Rev. Rel. **8**, 11 (2005); M. Bojowald, gr-qc/0505057.

[23] A. Ashtekar, M. Bojowald and J. Lewandowski, Adv. Theor. Math. Phys. **7**, 233 (2003) [gr-qc/0304074]; A. Ashtekar, gr-qc/0702030.

[24] A. Ashtekar, AIP Conf. Proc. **861**, 3 (2006) [gr-qc/0605011].

[25] C. Rovelli, Living Rev. Rel. **1**, 1 (1998) [gr-qc/9710008]; T. Thiemann, Lect. Notes Phys. **631**, 41 (2003) [gr-qc/0210094]; A. Corichi, J. Phys. Conf. Ser. **24**, 1 (2005) [gr-qc/0507038]; A. Perez, gr-qc/0409061.

[26] A. Ashtekar and J. Lewandowski, Class. Quant. Grav. **21**, R53 (2004) [gr-qc/0404018]; A. Ashtekar,

arXiv:0705.2222 [gr-qc].

- [27] C. Rovelli, Quantum Gravity, Cambridge University Press, Cambridge (2004). A. Ashtekar, New J. Phys. **7**, 198 (2005) [gr-qc/0410054]; T. Thiemann, hep-th/0608210.
- [28] A. Ashtekar, T. Pawłowski and P. Singh, Phys. Rev. Lett. **96**, 141301 (2006) [gr-qc/0602086]; Phys. Rev. D **73**, 124038 (2006) [gr-qc/0604013].
- [29] P. Singh and A. Toporensky, Phys. Rev. D **69**, 104008 (2004) [gr-qc/0312110]; G. V. Vereshchagin, JCAP **0407**, 013 (2004) [gr-qc/0406108]; G. Date and G. M. Hossain, Phys. Rev. Lett. **94**, 011302 (2005) [gr-qc/0407074].
- [30] M. Bojowald, Phys. Rev. Lett. **89**, 261301 (2002) [gr-qc/0206054]; M. Bojowald and K. Vandersloot, Phys. Rev. D **67**, 124023 (2003) [gr-qc/0303072]; M. Bojowald, J. E. Lidsey, D. J. Mulryne, P. Singh and R. Tavakol, Phys. Rev. D **70**, 043530 (2004) [gr-qc/0403106].
- [31] A. Ashtekar, T. Pawłowski and P. Singh, Phys. Rev. D **74**, 084003 (2006) [gr-qc/0607039].
- [32] P. Singh, Phys. Rev. D **73**, 063508 (2006) [gr-qc/0603043]; P. Singh, Class. Quant. Grav. **22**, 4203 (2005) [gr-qc/0502086].
- [33] Y. Shtanov and V. Sahni, Phys. Lett. B **557**:1 (2003)[gr-qc/0208047].
- [34] M. sami, P. Singh and S. Tsujikawa, Phys. Rev. D **74**, 043514 (2006); T. Naskar and J.Ward, arxiv: 0704.3606 [gr-qc].
- [35] D. Samart and B. Gumjupai, Phys. Rev. D **76**, 243514 (2007).
- [36] A. Ashtekar, A. Corichi and P. Singh, Phys. Rev. D **77**, 024046 (2008)[arxiv: 0710.3565]; A. Corichi and P. Singh, Phys. Rev. D **78**, 024034 (2008)[arxiv: 0805.0136].
- [37] P. X. Wu, S. N. Zhang, JCAP **06** (2008) 007.
- [38] X. Y Fu, H. W Yu and Puxun Wu, Phys. Rev. D **78**, 063001 (2008).
- [39] B. Gumjudpai, arXiv:0706.3467
- [40] H. Wei and S. N. Zhang, Phys. Rev. D **76**, 063005 (2007).
- [41] J.H. He, B. Wang, E. Abdalla, arXiv:0807.3471; J. Valiviita, E. Majerotto, R. Maartens, arXiv:0804.0232; P. S. Corasaniti, arXiv:0808.1646.